Vibration Damping

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Vibration Control by Damping

- Review of damping mechanisms and effects of damping
  - Viscous
  - Hysteretic
  - Coulomb

- Modelling and characteristics of viscoelastic damping

- Application of constrained and unconstrained layer damping

- Measurement of damping
Damping
dissipation of mechanical energy (usually conversion into heat)

- material damping (internal friction, mechanical hysteresis)
- friction at joints
- added layers of materials with high loss factors (e.g. visco-elastic materials)
- hydraulic dampers (shock absorbers, hydromounts)
- air/oil pumping: squeeze-film damping
- impacts (conversion of energy into higher frequency vibration which is then dissipated)
- acoustic radiation - vibration energy converted into sound (only important for lightly damped structures or heavy fluid loading, e.g. light plastic structures)
- structural ‘radiation’ - transport of vibration into adjacent structures or fluids
Viscous damping

The equation of motion is

\[ m\ddot{x} + cx + kx = 0 \]

- Inertia force
- Damping force
- Stiffness force
Example - automotive suspension system

- Connection part hinged eye
- Filling medium, gas
- Bottom valve unit
- Damping piston with valve system
- Pressure tube filled with oil
- Seal-guide unit
- Piston rod connection
Free vibration effect of damping

The \textit{underdamped} displacement of the mass is given by

\[ x = X e^{-\zeta \omega_n t} \sin \left( \omega_d t + \phi \right) \]

\( \zeta \) = Damping ratio = \( \frac{c}{2m \omega_n} \) \quad (0 < \zeta < 1)  

\( \omega_n \) = Undamped natural frequency = \( \sqrt{k/m} \)  

\( \omega_d \) = Damped natural frequency = \( \omega_n \sqrt{1 - \zeta^2} \)  

\( \phi \) = Phase angle
Free vibration effect of damping

\[ x = X e^{-\zeta \omega_n t} \]

\[ T_d = \frac{2\pi}{\omega_d} \]

\[ \zeta = \text{Damping ratio} \]

\[ T_d = \text{Damping period} \]

\[ \phi = \text{Phase angle} \]
Hysteretic Damping

Equation of motion

\[ m\ddot{x} + k(1 + j\eta)x = F_0 e^{j\omega t} \]

Assuming a harmonic response

\[ x = X e^{j\omega t} \]

leads to

\[ X = \frac{1}{k - \omega^2 m + jk\eta} \]

Setting responses to be equal at resonance gives

\[ \eta = 2\zeta \]

where \( \zeta = c/2\sqrt{mk} \)

**Hysteretic** damping force is *in-phase with velocity* and is *proportional to displacement*

**Viscous** damping force is *in-phase with and proportional to velocity*
Hysteretic Damping

Hysteretic damping model with a constant stiffness and loss factor is only applicable for **harmonic excitation**

\[ k(x + f) = F(t) \]

**causal**

\[ k(1 + j\eta)(x + f) = F(t) \]

**acausal**
Coulomb Damping

Coulomb damping is caused by sliding or dry friction.

\[ F_d = \text{friction force (always opposes motion)} \]

\[ F_d = -\mu F_n \quad \dot{x} > 0 \]
\[ 0 \quad \dot{x} = 0 \]
\[ \mu F_n \quad \dot{x} < 0 \]

- The damping force is independent of velocity once the motion is initiated, but the sign of the damping force is always opposite to that of the velocity.
- No simple expression exists for this type of damping. We can equate the energy dissipated per cycle to that of a viscous damper.

\[ F_n = mg \quad \text{is the reaction force} \]
How to compare damping mechanisms?

Energy dissipated by a viscous damper

\[ F_d = c \dot{x} \]

Energy lost per cycle is

\[ \Delta E = \text{work done} = \text{force} \times \text{distance} \]

\[ \Delta E = \int F_d \, dx = \int_0^T F_d \frac{dx}{dt} \, dt \]

\[ \Delta E = \int_0^{2\pi/\omega} c \dot{x}^2 \, dt \quad (1) \]

Now if

\[ x = X \sin(\omega t + \phi) \]

then \[ \dot{x} = \omega X \cos(\omega t + \phi) \]

Substitute into (1) gives

\[ \Delta E = c \omega^2 X^2 \int_0^{2\pi/\omega} \cos^2(\omega t + \phi) \, dt \]

which evaluates to give

\[ \Delta E = \pi c \omega X^2 \]
Returning to Coulomb Damping

- Since the damping force $F_d = \mu F_n$ is constant and the distance travelled in one cycle is $4X$, the energy dissipation per cycle is:

$$\Delta E = 4\mu F_n X$$

Noting that

$$\Delta E = \pi c_{eq} \omega X^2$$

Gives an equivalent viscous damping coefficient

$$c_{eq} = \frac{4\mu F_n}{\pi \omega X}$$

Note that this is a non-linear form of damping – dependent upon amplitude
Returning to Coulomb Damping

- Illustration of the non-linear effects of Coulomb damping
Coulomb Damping

- Illustration of the non-linear effects of Coulomb damping
Equivalent viscous damping forces and coefficients

<table>
<thead>
<tr>
<th>Damping Mechanism</th>
<th>Damping Force</th>
<th>Equivalent Viscous Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous</td>
<td>$c \dot{x}$</td>
<td>$C$</td>
</tr>
<tr>
<td>Hysteretic</td>
<td>$k\eta x$</td>
<td>$\frac{k\eta}{\omega}$</td>
</tr>
<tr>
<td>Coulomb</td>
<td>$\pm \mu F_n$</td>
<td>$\frac{4\mu F_n}{\pi \omega X}$</td>
</tr>
</tbody>
</table>
Forced vibration – effect of damping

\[ \frac{X}{F} = \frac{1}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \]

**Frequency Regions**

- **Low frequency** \( \omega \to 0 \) \( \Rightarrow \left| \frac{X}{F} \right| = 1/k \) **Stiffness controlled**
- **Resonance** \( \omega^2 = k/m \) \( \Rightarrow \left| \frac{X}{F} \right| = 1/\omega c \) **Damping controlled**
- **High frequency** \( \omega^2 \gg \omega_n \) \( \Rightarrow \left| \frac{X}{F} \right| = 1/m\omega^2 \) **Mass controlled**

Log \( \left| \frac{X}{F} \right| \) vs. Log frequency

Stiffness controlled \( \text{Damping controlled} \) Mass controlled
Example: point mobility of 1.5 mm steel plate $0.6 \times 0.5$ m

- Damping is only effective at controlling the response around the resonance frequencies.
- At high frequencies the mobility tends to that of an infinite structure and damping has no further effect.
Visco-elastic materials

- Visco-elastic materials (plastics, polymers, rubbers etc) have non-linear material laws.
- For a harmonic input, visco-elastic materials are defined in terms of their complex Young’s modulus $E(1+j\eta)$ or shear modulus $G(1+j\eta)$.
- Alternatively, we can write $E = E_R + jE_I$ where $E_R$ is called the storage modulus and $E_I$ is called the loss modulus.
- The parameters $E$, $G$ and $\eta$ are dependent on
  - frequency
  - temperature
  - strain amplitude
  - preload
- Frequency and temperature dependencies are equivalent (higher frequency is equivalent to lower temperature).

A model with a constant modulus and constant loss factor is often a good approximation in a limited frequency region. However, it gives non-causal response in the time domain.
Typical material damping

Loss Factor

Aluminum, Magnesium
Brass, Bronze, Steel, Iron
Copper, Tin
Lead
High-Damping Alloys

Metals

Glass
Brick, Masonry, Plaster on Lath Blocks

Building Materials

Gypsum Board
Oak, Fir Timber
Plywood, Particle Board
Dry Sand
Concrete (Light, Porous, Dense)
Asphalt

Building Materials

Plastics, Rubbers

Gels, Viscous Liquids
Visco-elastic damping – material properties

Simple model
• Combination of

Voigt model
Maxwell model

Dynamic stiffness is
\[
\frac{F}{X} = \frac{k_2}{c^2} + \frac{\omega^2}{k_1} \left( \frac{k_1 + k_2}{k_1} \right) + j \frac{\omega}{c}
\]

\[
\left( \frac{1}{c} \right)^2 + \left( \frac{\omega}{k_1} \right)^2
\]
Returning to Complex Stiffness

\[ F = k(\omega)(1 + j\eta(\omega)) \]

\[ \frac{F}{X} = k(\omega)(1 + j\eta(\omega)) \]

Stiffness determined from

\[ k(\omega) = \text{Re}\left\{ \frac{F}{X} \right\} \]

Loss factor determined from

\[ \eta(\omega) = \frac{\text{Im}\left\{ \frac{F}{X} \right\} \text{Re}\left\{ \frac{F}{X} \right\}}{\text{Im}\left\{ \frac{F}{X} \right\}} \]
Visco-elastic damping – material properties

Stiffness

\[
k(\omega) = \frac{k_2}{c^2} + \frac{\omega^2}{k_1} \left( \frac{k_1 + k_2}{k_1} \right) \left( \frac{1}{c} \right)^2 + \left( \frac{\omega}{k_1} \right)^2
\]

At low frequencies \( k(\omega) \to k_2 \)

At high frequencies \( k(\omega) \to k_1 + k_2 \)
Visco-elastic damping – material properties

Loss factor

\[ \eta(\omega) = \frac{\omega}{c} \left( \frac{k_2}{c^2} + \frac{\omega^2}{k_1} \left( \frac{k_1 + k_2}{k_1} \right) \right) \]

At low frequencies \( \eta(\omega) = \frac{\omega c}{k_2} \)

At high frequencies \( \eta(\omega) = \frac{1}{\frac{\omega c}{k_1} \left( \frac{k_1 + k_2}{k_1} \right)} \)
Visco-elastic materials

- Increasing frequency
- Increasing temperature

Rubbery region
Transition region
Glassy region

Stiffness and loss factor

Stiffness
Loss factor
Visco-elastic materials

(a) $E$ or $G$

(b) $\eta$ or $\beta$

Log frequency

Log frequency

SHIFT FACTOR, $\alpha_T$

$10^2$

$10$

$1$

$10^{-1}$

$10^{-2}$

$T_{-2}$ $T_{-1}$ $T_0$ $T_1$ $T_2$

TEMPERATURE
Visco-elastic materials – reduced frequency nomogram

Temperature

Modulus $E$ and loss factor $\eta$

Reduced frequency $f_{\alpha T}$, Hz

Frequency $f$, (Hz)
Visco-elastic materials – reduced frequency nomogram

- If the effects of damping and temperature on the damping behaviour of materials are to be taken into account, then the temperature-frequency equivalence (reduced frequency) is important.

- $E$ and $\eta$ are plotted against the reduced frequency parameter, $f\alpha_T$.

- To use the nomogram, for each specific $f$ and $T_i$ move down the $T_i$ line until it crosses the horizontal $f$ line. The intersection point $X$, corresponds to the value of $f\alpha_T$. Then move vertically up to read the values of $E$ and $\eta$. 
Visco-elastic materials – reduced frequency nomogram
Visco-elastic materials and damping treatments

- For damping treatments visco-elastic materials should generally be used in their transition region, where the loss factor is highest. However small changes in temperature can have a large change in stiffness.
## Typical material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus E (N m(^{-2}))</th>
<th>Density ρ (kg m(^{-3}))</th>
<th>Poisson’s ratio ν</th>
<th>Loss factor η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild Steel</td>
<td>2e11</td>
<td>7.8e3</td>
<td>0.28</td>
<td>0.0001 – 0.0006</td>
</tr>
<tr>
<td>Aluminium Alloy</td>
<td>7.1e10</td>
<td>2.7e3</td>
<td>0.33</td>
<td>0.0001 – 0.0006</td>
</tr>
<tr>
<td>Brass</td>
<td>10e10</td>
<td>8.5e3</td>
<td>0.36</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Copper</td>
<td>12.5e10</td>
<td>8.9e3</td>
<td>0.35</td>
<td>0.02</td>
</tr>
<tr>
<td>Glass</td>
<td>6e10</td>
<td>2.4e3</td>
<td>0.24</td>
<td>0.0006 – 0.002</td>
</tr>
<tr>
<td>Cork</td>
<td>1.2 – 2.4e3</td>
<td></td>
<td></td>
<td>0.13 – 0.17</td>
</tr>
<tr>
<td>Rubber</td>
<td>1e6 - 1e9</td>
<td>≈ 1e3</td>
<td>0.4 – 0.5</td>
<td>≈ 0.1</td>
</tr>
<tr>
<td>Plywood*</td>
<td>5.4e9</td>
<td>6e2</td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>Perspex</td>
<td>5.6e9</td>
<td>1.2e3</td>
<td>0.4</td>
<td>0.03</td>
</tr>
<tr>
<td>Light Concrete*</td>
<td>3.8e9</td>
<td>1.3e3</td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td>Brick*</td>
<td>1.6e10</td>
<td>2e3</td>
<td></td>
<td>0.015</td>
</tr>
</tbody>
</table>
Damping treatments using visco-elastic materials
Damping treatments using visco-elastic materials
Unconstrained layer damping

- Applied to one free surface
- Loss due to extension of damping material ($E\eta$ product is important)
- Frequency independent (apart from material properties)
Unconstrained layer damping

- The strain energy in a uniform beam of length $l$ is given by

$$U = \frac{1}{2} EI \int_0^l \left( \frac{d^2 y}{dx^2} \right)^2 dx$$

(1)
Unconstrained layer damping

- A definition of the loss factor is

\[ \eta = \frac{1}{2\pi} \times \frac{\text{energy dissipated per cycle}}{\text{maximum energy stored per cycle}} \]

Let \( \eta_b \) = loss factor for the composite structure

Let \( \eta_d \) = loss factor for the damping layer

Assume that the loss factor in the structure is negligible compared with the damping layer

\[ \frac{\eta_b}{\eta_d} = \frac{\text{energy dissipated per cycle in the composite structure}}{\text{maximum energy stored per cycle in the composite structure}} \div \frac{\text{energy dissipated per cycle in the damping layer}}{\text{maximum energy stored per cycle in the damping layer}} \]
Unconstrained layer damping

So \[ \frac{\eta_b}{\eta_d} = \frac{\text{maximum energy stored per cycle in the damping layer}}{\text{maximum energy stored per cycle in the composite structure}} \]

Substitute from (1) gives

\[ \frac{\eta_b}{\eta_d} = \frac{E_d I_d}{E_d I_d + E_s I_s} \] (I’s are about the composite neutral axis)

Thin damping layers

\[ \frac{\eta_b}{\eta_d} \approx \frac{E_d I_d}{E_s I_s} \]

Thus it is not sufficient to have a large \( \eta_d \); a large \( E_d \eta_d \) is also required

Thick damping layers

\[ \eta_b \rightarrow \eta_d \]

To be effective damping layers of similar thickness to the structure are required

\[ \text{Weight penalty} \]
Unconstrained layer damping

\[ \frac{\eta_b}{\eta_d} = \frac{1}{1 + \frac{E_s l_s}{E_d l_d}} \]

In this region the damping layer also affects the overall stiffness of the structure.

i.e. require:
- \( E_d \) large
- \( h_2 \) large
- \( \eta_d \) large
Constrained layer damping

- Sandwich construction
- Loss due to shear of damping layer ($G\beta$ product is important)
- Constraining layers must be stiff in extension
- Thin damping layers can give high losses
- Frequency dependent properties
Constrained layer damping

$\eta_{composite}/\beta$ depends on two non-dimensional parameters:

- ‘geometric parameter’ (also depends on elastic moduli)

$$L = \frac{(E_1 h_1)(E_3 h_3)d^2}{(E_1 h_1 + E_3 h_3)(E_1 I_1 + E_3 I_3)}$$

- here $I_1$ and $I_3$ are relative to own centroids
- large for deep core, small for thin core ($d$)

- ‘shear parameter’

$$g = \frac{G}{d^2 h_k} \left( \frac{1}{E_1 h_1} + \frac{1}{E_3 h_3} \right)$$

- large for low frequencies, small for high frequencies

$\begin{array}{|c|c|}
\hline
h_1 : h_1 : h_3 & L \\
1 : 0.1 : 0.1 & 0.46 \\
1 : 0.1 : 1 & 3.63 \\
1 : 1 : 1 & 12 \\
1 : 10 : 1 & 363 \\
\hline
\end{array}$

$E_1 = E_3$
**Constrained layer damping**

\[
\eta_{\text{composite}} = \frac{\beta}{\eta_{\text{composite}}}
\]

In this region large \( L \) means much stiffer structure.

The shear parameter \( g \) is defined as:

\[
g = \frac{G}{h_2k_b^2} \left( \frac{1}{E_1h_1} + \frac{1}{E_3h_3} \right)
\]

Where:
- \( G \) is the shear modulus of the composite material.
- \( h_2 \) is the thickness of the second layer.
- \( k_b \) is the wavelength of the wave.
- \( E_1, E_3 \) are the Young's moduli of the composite layers.
- \( h_1, h_3 \) are the thicknesses of the respective layers.

The damping parameter \( \eta \) is given by:

\[
L = \frac{(E_1h_1)(E_3h_3)d^2}{(E_1h_1 + E_3h_3)(E_1l_1 + E_3l_3)}
\]
Constrained layer damping

Effectiveness also depends on loss factor of damping material

At **low frequencies** (high temp) constrained layer damping is ineffective because the damping material is operating in its rubbery region and is thus soft. The structure and the constraining layer become uncoupled.

At **high frequencies** (low temp) constrained layer damping is ineffective because the damping material is operating in its glassy region and is thus hard. The structure and the constraining layer tend to move as one.
Other types of damping

Damping by air pumping

Viscous damping – reduces in effectiveness at high frequencies

Pumping along joints
Damping is achieved by air being pumped between the main plate and the attached plate (JSV 118(1), 123-139, 1987)
Squeeze-film damping II

- Oil film is squeezed between the bearing and the housing generating damping forces
Summary: means of adding damping

- unconstrained layer damper
- constrained layer damper (can be multiple layers)
- friction damping: important at bolted or rivetted joints.
- squeeze film damping
- impact damper
- change in structural material.
- tuned vibration absorber (added damped mass-spring systems)
Measurement of Damping

- Decay of free vibration measurement
- Response curve at resonance measurement
- Complex modulus measurement
Measurement of Damping – decay of free vibration

\[ x(t) = X e^{-\zeta \omega_n t} \]

Measure the amplitude of two successive cycles

\[ \frac{x_1}{x_2} = e^{\zeta \omega_n T_d} \]

Logarithmic decrement

\[ \delta = \ln \left( \frac{x_1}{x_2} \right) = \zeta \omega_n T_d \]

So

\[ \delta = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}} \]

For light damping \( \delta \approx 2\pi\zeta \)

So

\[ \zeta \approx \frac{\delta}{2\pi} \]

measured
Measurement of Damping – decay of free vibration

- In practice it is better to measure the logarithmic decrement over a number of cycles

\[ \delta = \frac{1}{n} \ln \left( \frac{x_i}{x_{i+n}} \right) \]

\( n \) can be determined by knowledge of the natural frequency and the time between \( x_i \) and \( x_{i+n} \)
Measurement of Damping – response curve at resonance

Response curve

\[ \frac{X}{F} \]

Frequency

Nyquist plot

\[ \frac{\text{Re}\left\{ \frac{X}{F} \right\}}{\text{Im}\left\{ \frac{X}{F} \right\}} \]

\[ \omega_1, \omega_n, \omega_2 \]

\[ \zeta = \frac{\omega_2 - \omega_1}{2 \omega_n} \]
Measurement of Damping – complex modulus measurement

Impedance head
(measures force and acceleration)

very stiff, light material

specimen
area $A$

Measure dynamic stiffness

$$\frac{F}{X} = k(1 + j\eta)$$

Stiffness
$$k = \text{Re} \left\{ \frac{F}{X} \right\}$$

Loss factor
$$\eta = \frac{\text{Im} \left\{ \frac{F}{X} \right\}}{\text{Re} \left\{ \frac{F}{X} \right\}}$$

Compute $E$ by using
$$k = \frac{EA}{l}$$
Summary

• Review of damping mechanisms

• Modelling and characteristics of viscoelastic damping

• Application of damping treatments

• Measurement of damping
References