Sources of Vibration

Professor Mike Brennan
Sources of Vibration

- Rotating machines
- Gears
- Acoustic excitation
- Impacts
- Vortex shedding
- Flutter
- Self-excited vibration
Impact as a source of vibration

- Impacts occur in many engineering situations. As they take place over a very short time-period, the frequency content of the impacting force tends to be rich. Some examples are illustrated below.
**Impact as a source of vibration**

**Example**

- Consider an elastic sphere that strikes a *rigid* surface as shown below.

  ![Diagram](image)

- Once the mass-spring system is in contact with the surface it will begin to oscillate with a period
  \[ T = 2\pi \sqrt{\frac{m}{k}} \]

- The displacement will be *sinusoidal during the half-period* the body stays in contact with the surface.

- The transmitted force is the product of the displacement and the contact stiffness. Thus there is a a *half-sine force-time curve.*
Impact as a source of vibration

Time-history

Energy spectrum

- Contact time $T/2 = \pi \sqrt{m/k}$

- To increase the contact time the stiffness of the impacting body should be decreased. The high frequency content is then reduced.
Vibrations generated by rotating machinery

Gears

• The driving wheel is assumed to rotate at constant angular velocity $\Omega_1$ and the driven wheel has an **average** angular velocity $\Omega_2$, and has angular displacement

$$\theta_2 = \Omega_2 t + \Delta \theta$$

where $\Omega_2 = \frac{r_1}{r_2} \Omega_1$ and $\Delta \theta$ is known as the **transmission error**

Transmission error is defined as the difference between the actual position of the output gear and the position it would take if it were perfect.
Vibrations generated by rotating machinery

Gears

- transmission error occurs due to
- deflection of gear teeth under load
- manufacturing defects

- the fundamental meshing frequency of the meshing force is given by

\[ \text{fundamental meshing frequency} = \Omega \times \text{number of teeth} \]
Vibrations generated by rotating machinery

Gears

Typical spectrum

![Graph showing vibrations](image)
Vibrations generated by rotating machinery

Rotating unbalance

In a rotating machine, unbalance exists if the mass centre of the rotor does not coincide with the axis of rotation.

The unbalance $me$ is measured in terms of an equivalent point mass $m$ with an eccentricity $e$.

The equation of motion is given by

$$m_t \ddot{x} + c \dot{x} + kx = me \omega^2 \sin \omega t$$

Assuming a harmonic response $x = X \sin \omega t$ results in

$$X = \frac{me \omega^2}{k} \left| \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} + j \frac{2 \zeta \omega}{\omega_n} \right|$$

where $\omega_n = \sqrt{k/m_t}$ and $\zeta = c/2 \sqrt{km_t}$
Vibrations generated by rotating machinery

Rotating unbalance

\[ \frac{Xm_t}{me} = \left( \frac{\omega}{\omega_n} \right)^2 \left| \frac{1}{1 - (\omega/\omega_n)^2 + j2\zeta \omega/\omega_n} \right| \]

- At low speeds \( \omega/\omega_n \) is small and hence the vibration amplitude is also small
- At resonance when \( \omega/\omega_n = 1 \) damping controls the vibration amplitude
- At high speeds when \( \omega/\omega_n >> 1 \), \( X = me/m_t \), which means that the vibration is constant and is independent of speed
The excitation force is equal to the inertia force of the reciprocating mass, which is approximately equal to

\[ \text{dynamic force} \approx me \omega^2 \left( \sin \omega t + \frac{e}{L} \sin 2\omega t \right) \]

Note the second harmonic.
Vibrations generated by rotating machinery

Critical speed of a rotating shaft

- Mass of the shaft is negligible
- Lateral stiffness of the shaft is \( k \)
- \( M \) is the mass of the rotating disk
- \( G \) is the mass centre of the disk
- \( S \) is the geometric centre of the disk
- The bearings are much stiffer than the shaft

The **critical** speed occurs when the speed of rotation of the shaft is equal to the frequency of **lateral** vibration of the shaft
Vibrations generated by rotating machinery

Critical speed of a rotating shaft

The equations of motion in the $x$ and $y$ directions are

$$m_t \ddot{x} + c \dot{x} + kx = me\omega^2 \sin \omega t$$

$$m_t \ddot{y} + c \dot{y} + ky = me\omega^2 \cos \omega t$$

Assuming a harmonic response gives

$$\frac{Xm_t}{me} = \left( \frac{\omega}{\omega_n} \right)^2 \left| 1 - (\omega/\omega_n)^2 + j2\zeta \omega/\omega_n \right|$$

and

$$Y = Xe^{j\pi/2}$$

Since the two harmonic motions $x(t)$ and $y(t)$ have the same amplitude, and are in quadrature, **their sum a circle**. The radius $r = X$ or $Y$
Vibrations generated by rotating machinery

Critical speed of a rotating shaft – effects of bearing stiffness

- The stiffness of the bearings acts in series with the stiffness of the shaft. Thus the system becomes more flexible lowering the critical speed
Vibrations generated by rotating machinery

Critical speed of a rotating shaft – effects of bearing stiffness

The equations of motion in the $x$ and $y$ directions are

$$m_t \ddot{x} + c \dot{x} + k_x x = me\omega^2 \sin \omega t$$

$$m_t \ddot{y} + c \dot{y} + k_y y = me\omega^2 \cos \omega t$$

If $k_x < k_y$ the system has two natural frequencies and therefore two critical speeds. Since the two harmonic motions $x(t)$ and $y(t)$ have different amplitudes, and are in quadrature, their sum an ellipse.
λ is the wavelength of the impinging sound field

λ<sub>t</sub> is the **trace wavelength** and is given by \( \lambda_t = \lambda / \sin \theta \)

The trace velocity is the speed at which the wavefront moves along the beam and is given by \( c_t = c / \sin \theta \) where \( c \) is the speed of sound in air

The pressure distribution on the beam is given by

\[
p(x,t) = Pe^{i(\omega t - k_t x)}
\]

where \( k_t \) is the trace wavenumber given by

\[
k_t = \frac{2\pi}{\lambda_t} = \frac{\omega}{c_t}
\]
Acoustic Excitation

The maximum response of the beam occurs at

**A RESONANT FREQUENCY**, when the length of the beam is equal to an integer number of half-wavelengths

**A COINCIDENT FREQUENCY**, when the trace wavelength is equal to a flexural wavelength
Acoustic Excitation

Speed of sound

Critical frequency

Speed of bending waves

Bending waves

Sound waves
Acoustic Excitation

Critical frequency

The critical frequency $f_c$ is the lowest frequency at which coincidence can occur.

It is a frequency when the speed of the impinging sound wave $c$, equals the speed of the flexural bending wave $c_b$.

The critical frequency for a plate can be determined using the equation

$$f_c = \frac{c^2}{1.8hc_l}$$

where $c_l$ is the longitudinal wavespeed ($\approx5000$ ms$^{-1}$ for steel and aluminium) and $h$ is the plate thickness

<table>
<thead>
<tr>
<th>Material</th>
<th>$hf_c$ (ms$^{-1}$)</th>
</tr>
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<tbody>
<tr>
<td>Steel</td>
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</tr>
<tr>
<td>Aluminium</td>
<td>12.0</td>
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<tr>
<td>Brass</td>
<td>17.8</td>
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<tr>
<td>Copper</td>
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<td>Glass</td>
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<td>Perspex</td>
<td>27.7</td>
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<tr>
<td>Chipboard</td>
<td>23</td>
</tr>
<tr>
<td>Plywood</td>
<td>20</td>
</tr>
<tr>
<td>Light concrete</td>
<td>34</td>
</tr>
</tbody>
</table>

* For water multiply by 18.9
The *coincidence angle* $\theta_{co}$, is given by

$$
\theta_{co} = \sin^{-1}\left(\frac{\omega}{\omega_c}\right)^\frac{1}{2}
$$

**Acoustic Excitation**

**Coincidence angle**

![Graph showing the relationship between $\theta_{co}$ and $\omega/\omega_c$ with a critical frequency and no coincidence region.](chart.png)
Self-excited vibrations

The *force* acting on a vibrating system is usually *independent* of the motion of the system.

In a self-excited system the *excitation force is a function of the motion of the system*.

Examples of such systems where this phenomenon occurs are

- Instabilities of rotating shafts
- The flutter of turbine blades
- The flow induced vibration of pipes
- Aerodynamically induced motion of bridges
- Brake squeal
Self-excited vibrations

Stable system

Unstable system

Vibrating system

Feedback system
Self-excited vibrations

To see the circumstances that lead to instability, consider a SDOF system

The equation of motion is

\[ m\ddot{x} + c\dot{x} + kx = 0 \]

Substituting for \( \omega_n^2 = \frac{k}{m} \) and \( \zeta = \frac{c}{2\sqrt{mk}} \) gives

\[ \ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = 0 \]

Assuming a solution of the form \( x = X e^{st} \) gives the characteristic equation

\[ s^2 + 2\zeta \omega_n s + \omega_n^2 = 0 \]

which has a solution

\[ s_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \]

For oscillatory motion \( s_{1,2} \) are complex. The system will be dynamically Stable provided \( \zeta \geq 0 \)
Self-excited vibrations

Locus of $S_1$ and $S_2$

$\zeta > 1$

$0 < \zeta < 1$

$\zeta = 1$

$\zeta = 0$

$\zeta < 0$

stable

unstable

imaginary axis

real axis
Self-excited vibrations

Simple example

The vibration of an aircraft wing can be crudely modelled as

\[ m\ddot{x} + c\dot{x} + kx = Ax \]  \hspace{1cm} (1)

where \( m, c \) and \( k \) are the mass, damping and stiffness values of the wing modelled as a SDOF system and where \( Ax \) is an approximate model of the aerodynamic force on the wing.

Rearranging (1) gives

\[ m\ddot{x} + (c - A)\dot{x} + kx = 0 \]

If \( C > A \) the system is stable \hspace{1cm} \text{If } C < A \text{ the system is unstable}

This is an example of flutter instability
Flow induced vibrations

There are three mechanisms for inducing vibration by fluid flow

- Turbulence
- Vortex shedding
- Instabilities
Vibrations induced by turbulence

- Most common cause of flow induced vibration
  - swaying of trees by gusts of wind
  - vibration of a car aerial due to turbulent flow
  - anchored ship bobbing in a turbulent sea
  - heat exchanger vibration due to turbulent flow
  - vibration of aircraft fuselage due to turbulent boundary layer

- random vibration problem

\[ S_{gg} = |H(j\omega)|^2 S_{ff} \]
Vortex induced vibration

- Note vortices being shed alternatively from each side of the structure
Vortex induced vibration

- Any structure with sufficiently bluff trailing edge sheds vortices in a subsonic flow

- The separation of the flow is a function of the Reynolds number \( (Re) \)

\[
Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{uD}{v}
\]

- \( u \) is the flow velocity
- \( D \) is width normal to the free stream
- \( v \) is the kinematic viscosity of the fluid which is the viscosity divided by the fluid density
Vortex induced vibration regimes of flow across a circular cylinder [after Blevins]

\[
\begin{align*}
\text{Re} &< 5 \quad \text{REGIME OF UNSEPARATED FLOW} \\
5 \text{ TO } 15 &\leq \text{Re} < 40 \quad \text{A FIXED PAIR OF FÖPPL VORTICES IN WAKE} \\
40 &\leq \text{Re} < 90 \text{ AND } 90 &\leq \text{Re} < 150 \quad \text{TWO REGIMES IN WHICH VORTEX STREET IS LAMINAR}
\end{align*}
\]
Vortex induced vibration regimes of flow across a circular cylinder [after Blevins]

150 \leq \text{Re} < 300 \quad \text{TRANSITION RANGE TO TURBULENCE IN VORTEX}

300 \leq \text{Re} \lesssim 3 \times 10^5 \quad \text{VORTEX STREET IS FULLY TURBULENT}

3 \times 10^5 \leq \text{Re} < 3.5 \times 10^6 \quad \text{LAMINAR BOUNDARY LAYER HAS UNDERGONE TURBULENT TRANSITION AND WAKE IS NARROWER AND DISORGANIZED}

3.5 \times 10^6 \leq \text{Re} \quad \text{RE-ESTABLISHMENT OF TURBULENT VORTEX STREET}
Frequency of vortex shedding

- The frequency of vortex shedding $f_s$ is given by

$$f_s = \frac{Su}{D}$$

$u$ is the flow velocity

$D$ is width normal to the free stream
Vortex induced vibration – other effects

Self-Limiting Effect

If the amplitude of vibration increases beyond approximately $0.5D$, the symmetric pattern of alternatively spaced vortices begins to break up. This means that the vortex induced forces are self-limiting at vibrations of the order of $D$.

Lock-In Effect

When the vortex shedding frequency is close to the cylinder natural frequency, the frequency of vortex shedding can shift to the cylinder natural frequency. This is known as the lock-in effect.
Vortex induced vibration - solutions

- Increase internal damping

- Avoid resonance \( u < \frac{f_n D}{S} \)
  
  where \( f_n \) is the natural frequency of the structure

- Take measures to eliminate vortex shedding
Galloping vibrations and stall flutter

If a structure vibrates in a **steady flow**, the flow in turn oscillates relative to the moving structure, and acts as an oscillating aerodynamic force on the structure.

If this force tends to diminish the vibrations of the structure, then the structure is said to be **aerodynamically stable**.

If this force tends to increase the vibrations of the structure, then the structure is said to be **aerodynamically unstable**.

**Examples**

**Flutter** – aircraft wings

**Galloping** – ice coated power lines

![Diagram of flutter in aircraft wing](image1)

![Diagram of galloping in ice coated power lines](image2)
Example - flutter
Galloping vibrations and stall flutter

Example

The vertical force acting on the aerofoil due to the fluid flow is given by [Blevins]

\[
f = \frac{1}{2} \rho u^2 D c_L - \frac{1}{2} \rho u D \left( \frac{\partial c_L}{\partial \alpha} + c_D \right) \dot{x}
\]

- \( \rho \) is the density of the fluid
- \( c_L \) is the lift coefficient
- \( c_D \) is the drag coefficient
- \( u \) is the free stream velocity
Galloping vibrations and stall flutter

The equation of motion is

\[ m\ddot{x} + c\dot{x} + kx = f \]

Substituting for the force component related to velocity gives

\[ m\ddot{x} + c\dot{x} + kx = -\frac{1}{2} \rho u D \left( \frac{\partial c_L}{\partial \alpha} + c_D \right) \dot{x} \]

or

\[ m\ddot{x} + \left( c + \frac{1}{2} \rho u D \left( \frac{\partial c_L}{\partial \alpha} + c_D \right) \right) \dot{x} + kx = 0 \]

Unstable when this \(< 0\)

Thus the minimum flow velocity to cause instability is

\[ u = \frac{-2c}{\rho D \left( \frac{\partial c_L}{\partial \alpha} + c_D \right)} \]

The system is unstable if the lift force increases when the model is slightly rotated in a steady wind such that \( u = \frac{\partial c_L}{\partial \alpha} + c_D < 0 \)
Galloping vibrations and stall flutter

\[ \frac{\partial C_x}{\partial \alpha} = \frac{\partial C_L}{\partial \alpha} + C_D \]

\[ C_x = \text{vertical force coefficient} \]
Example – Tacoma Narrows Bridge
Example – bridge vibrations

Profile of Tacoma Narrows bridge

unstable

structural damping

stable

Profile of Severn bridge

\[ u_r = \frac{u}{f_T \cdot b} \]
Noise from Vortices

Vortices propagate at speed of the flow

Vortices impinge on structure generating sound

Sound propagates at speed of sound
Noise from Vortices

Increasing flow speed
Summary

The following sources of vibration have been discussed:

- Impacts
- Rotating machinery
  - Gears
  - Mass unbalance
- Acoustic excitation
- Self-excited vibration
- Flow-induced vibration
References


