Duct acoustics
WHY STUDY THE ACOUSTICS OF DUCTS?

Ducts, also known as waveguides, are able to efficiently channel sound over large distances. Some common examples are:

- Ventilation Ducts
- Exhaust Stacks
- Automotive Silencers (Mufflers)
- Aircraft Turbofan Engines
- Shallow Water Channels and Surface Ducts in Deep Water
TYPES OF DUCT SILENCERS

- Sound radiated from duct systems can be reduced by the use of reactive or dissipative silencers.

- Reactive silencers – the transmitted sound power is reduced by reflecting some of the incident sound power (usually by changes in the geometry of the duct).

- Dissipative silencers – the transmitted sound power is reduced by dissipating some of the incident sound power as heat.
REACTIVE SILENCERS
PLANE WAVE PROPAGATION
Sound Waves in a Duct

\[ P = \frac{F}{S} \]

\[ Ve^{j\omega t} \]

\[ Fe^{j\omega t} \]

\[ \text{Area} = S \]

Acoustic Impedance \( Z_a \) = \( \frac{P}{V} = \rho_0 c \)
Sound Waves in a Duct

\[ F e^{j\omega t} \quad \rightarrow \quad A e^{j(\omega t - kx)} \quad \rightarrow \quad B e^{j(\omega t + kx)} \]

\[ k = \frac{\omega}{c} \quad \text{is the wavenumber} \]

\[ P(x) = A e^{-jkx} + B e^{jkx} \]
The Wavenumber (propagating waves)

$$\omega = \frac{2\pi}{T}$$

Temporal frequency

$$k = \frac{2\pi}{\lambda}$$

Spatial frequency (wavenumber)

$k$ is the phase change per unit distance

$$Ae^{-jkx}$$
Sound Waves in a Duct

\[ F e^{j \omega t} \rightarrow \quad A e^{j(\omega t - kx)} \rightarrow \quad B e^{j(\omega t + kx)} \]

\[ P(x) = A(e^{-j kx} + R e^{j kx}) \]

where \( R \) is the reflection coefficient
Sound Waves in a Duct

\[
\frac{P}{\rho_0 c V} = \frac{Z_a}{\rho_0 c} = \frac{A e^{jkL} + B e^{-jkL}}{A e^{jkL} - B e^{-jkL}}
\]

Or in terms of an impedance $Z_L$ at the end

\[
\frac{Z_a}{\rho_0 c} = \frac{Z_L}{\rho_0 c} + j \tan kL
\]

\[
1 + j \frac{Z_L}{\rho_0 c} \tan kL
\]
Closed Duct

\[ Z_a = \frac{1}{\rho_0 c \left( j \tan kL \right)} \]

\[ L \lambda \]
$$\frac{Z_a}{\rho_0 c} = j \tan kL$$
Intake and Exhaust Noise
Intake and Exhaust Noise
Muffler Performance

Insertion loss
\[ IL = L_{p2} - L_{p1} \]

Transmission loss
\[ TL = 10 \log_{10} \frac{S_iI_i}{S_tI_t} \]

Noise reduction
\[ NR = L_{p1} - L_{p2} \]
Acoustic Filters

\[ \hat{s} = \frac{S}{S_p} \]

\[ TL = 10 \log \left| 1 + \frac{\hat{s} \rho_0 c}{2Z} \right|^2 \]
Acoustic Filters – High-Pass

\[
\frac{L'}{\lambda} = \frac{\hat{s}}{4\pi}
\]

\[
\hat{s} = 1
\]

\[
TL = 10\log\left(1 + \left(\frac{\hat{s}}{2kL'}\right)^2\right)
\]

\[
S = \pi a^2
\]

\[
\hat{s} = \frac{S}{S_p}
\]

\[
L' = L + 1.5a
\]
Acoustic Filters – Quarter Wave Resonator

\[\hat{S} = 1\]

\[TL = 10\log\left(1 + \left(\frac{\hat{S}\tan kL}{2}\right)^2\right)\]

\[\frac{L}{\lambda}\]

\[S = \pi a^2\]

\[S_p\]

\[\hat{S} = \frac{S}{S_p}\]
Acoustic Filters – Helmholtz Resonator

\[ S = \pi a^2 \]

\[ L' = L + 1.7a \]

\[
\alpha = \left( \frac{L'S_p^2}{VS} \right)^{\frac{1}{4}}
\]

\[
TL = 10\log \left( 1 + \frac{\hat{\omega}^2}{4\alpha \left( \hat{\omega}^2 - 1 \right)^2} \right)
\]
Acoustic Filters – Band-Pass

\[ \hat{s} = 10 \]

\[ TL = 10 \log \left( \cos^2 kL + \frac{1}{4} \left( \hat{s} + \frac{1}{\hat{s}} \right)^2 \sin^2 kL \right) \]

\[ TL_{\max} = 10 \log \left( \frac{1}{4} \left( \hat{s} + \frac{1}{\hat{s}} \right)^2 \right) \]

\[ \hat{s} = \frac{S_c}{S_p} \]
Mufflers
Muffler Performance
- effect of absorption

(a) $r_2 = r_1$

(b) $r_2 = 3r_1$

(c) $r_2 = 5r_1$

Graph:
- $r_1 = 1.98$ cm
- $l = 32.5$ cm
- $th = 1.82$ cm
- FR = 5000 pa s/m²

TL_a (dB)

Frequency (kHz)
## Acoustic Filters

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<tr>
<th>Attenuation element</th>
<th>Attenuation characteristics</th>
<th>Comment</th>
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<tr>
<td>Packed (dissipative)</td>
<td>Broad band</td>
<td>Low backpressure</td>
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<td>Expansion chamber</td>
<td>att</td>
<td>Max attenuation $\propto \left(\frac{d_2^2}{2d_1^2}\right)^2$</td>
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<tr>
<td>1/4 Wave side branch</td>
<td>Narrow band (with odd order harmonics)</td>
<td>More commonly used for higher frequencies</td>
</tr>
<tr>
<td>Helmholtz resonator</td>
<td>Narrow band</td>
<td>More commonly used for lower frequencies</td>
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<tr>
<td>Cross flow and perforates</td>
<td>Various but generally broad band</td>
<td>Prone to high-frequency gas rush noise at high flows</td>
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HIGHER-ORDER MODE PROPAGATION
MODES AND MODE SHAPE FUNCTIONS

At frequencies greater than \( f > 1.84 \frac{c}{\pi D} \), higher order modes can propagate. The general solution is a linear superposition of these ‘mode shape function’ solutions:

**Cylindrical duct**

\[
\hat{p}(r, \theta, x) = \sum_{m=\infty}^{\infty} \sum_{n=1}^{\infty} A_{mn} \Phi_{mn}(r, \theta)e^{-i\alpha_{mn}kx}
\]

\[
\Phi_{mn}(r, \theta) = J_m(\kappa_{mn}r)e^{-im\theta}
\]

**Rectangular duct**

\[
\hat{p}(y, z, x) = \sum_{ny=1}^{\infty} \sum_{nz=1}^{\infty} A_{nyz} \Phi_{nyz}(y, z)e^{-i\alpha_{nyz}kx}
\]

\[
\Phi_{nyz}(y, z) = \cos(k_{ny}y)\cos(k_{nz}z)
\]

The resultant acoustic pressure in the duct is the weighted sum of fixed pressure patterns across the duct cross section. Each of which propagate axially along the duct at their characteristic axial phase speeds.
EXAMPLES – RIGID CIRCULAR-DUCT MODES

(0,4)  (2,2)  (2,4)

(4,3)  (8,2)  (12,1)
Wave characteristics

Flexural structural waves

\[ c \propto \omega^2 \]

Acoustic plane waves

\[ c \propto \omega \]
RESISITIVE SILENCERS
TYPES OF ACOUSTIC LINER

- Single degree of freedom (SDOF) cavity liner
- Two degree of freedom (2DOF) cavity liner
- Bulk absorber liner
TYPES OF ACOUSTIC LINER

- Partitions (honeycomb)
  - Porous face-sheet
  - Rigid backplate
  - Single degree of freedom (SDOF) liner

- Porous septum
  - Partitions (honeycomb)
  - Porous face-sheet
  - Rigid backplate
  - Two degree of freedom (2DOF) liner

- Fibrous material
  - Porous face-sheet
  - Wave propagation
  - Rigid backplate
  - Bulk absorber liner
EXAMPLE OF A PRACTICAL LINER

(Photograph of a Rolls-Royce fan rig, from AIAA paper no. 2001-2268)
Cavity liners (SDOF, 2DOF etc) consist of a sandwich construction. Each layer consists of a porous sheet and a cellular separator such as honeycomb. The backplate is rigid.

Cavity liners can be tuned to the frequency band of interest (by varying the partition depth). The bandwidth can be extended by the addition of more layers.

The acoustic properties of cavity liners depend on the resistance of the porous sheet(s), and the depth of the partition(s).

Cavity liners are “locally-reacting” liners, because the cellular structure prevents lateral sound propagation within the lining.
BULK ABSORBERS

- Bulk absorber liners are a single layer construction. A fibrous material (e.g. glass fibre, mineral wool etc) fills the gap between a porous face-sheet and a rigid back plate.

- Bulk absorbers tend to have the widest bandwidth, and are best suited to absorb broadband and low-frequency noise.

- Bulk absorbers are “bulk-reacting” liners, because sound can be transmitted within the lining.
SPECIFIC ACOUSTIC IMPEDANCE

- Specific acoustic impedance $z$, at a point in a single-frequency sound field, is the complex ratio

$$z = \frac{\hat{p}}{\hat{u}}$$

Units: Pa s m$^{-1}$ or SI rayl

- At a boundary, the acoustic particle velocity is the component normal to the wall. Acoustic pressure – $\hat{p}e^{i\omega t}$, Acoustic particle velocity – $\hat{u}e^{i\omega t}$

- It is convenient to nondimensional the value of $z$ as follows:

$$Z = \frac{z}{\rho_0 c_0} = R + iX$$

$R$ – Resistance

$X$ – Reactance

- $\rho_0 c_0$ is the characteristic impedance of the fluid (e.g. air).
CREMER OPTIMUM IMPEDANCE - EXAMPLE

$\Delta_{mn}$ contour map

$\Delta_{mn}$, $m = 0$, $n = 1$

$kb = 10$  $M_x = 0$

Optimum impedance

$Z = 2.95 - 1.25i$
\[ k_{x_{mn}} b \quad m = 0 \text{ only} \]

\[ kb = 10 \quad M_x = 0.4 \]

Propagating (cut-on) modes

\[-kM_x / (1 - M_x^2)\]
EIGENVALUES – lined duct

\[ k_{x,mn} b \quad m = 0 \text{ only} \]

\[ kb = 10 \quad M_x = 0.4 \]

\[ Z = 1.0 - 0.5i \]
EXAMPLES: LINED-DUCT MODES

(0,1) (2,1) (4,1)

(6,1) (8,1) (10,1)
The transmission loss of mode \((m,n)\) across a lined section of length \(l\), in decibels, is

\[
\Delta_{mn} = 20 \log_{10} \left| \frac{\hat{p}(x = 0)}{\hat{p}(x = l)} \right| = -20 \text{Im}\{k_{xmn}\} l \log_{10} e \\
\approx 8.6859 \alpha_{mn} l
\]

The attenuation of a mode \((m,n)\) is proportional to \(\alpha_{mn}\).

(The axial decay rate \(- \text{Im}\{k_{xmn}\}\) is denoted by \(\alpha_{mn}\).)
LINER ATTENUATION VERSUS MODAL PROPAGATION ANGLE

Modes near cut-off – well absorbed

Modes not near cut-off – poorly absorbed
VISUALISATION OF PRESSURE FIELD